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2d Gravity and the Black Hole Solution
in 2d Critical String Theory

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ABSTRACT

We discuss the relation between 2d gravity and critical string theory in two target space dimensions. In particular we consider the effect that the tachyon, and its back reaction on the metric, has on the interpretation of the critical theory as a noncritical theory coupled to world sheet gravity. We examine the generalizations of the black hole solution of the critical theory and show that the Hawking temperature is independent of the additional parameters. We argue that a generic feature of these solutions is that they have two horizons, similar to, but distinct from, the Reissner-Nordstrom black hole. We also find some indication that the tachyon may destabilize a naked singularity. Finally we show that KPZ scaling is valid for the general solution of the critical theory, when it is interpreted as a theory of world sheet gravity.

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It is by now commonplace to consider 2d gravity coupled to $c = 1$ matter as a critical string theory in two target space dimensions [1] in a particular background. This point of view has been used most effectively to explain the logarithmic scaling violations of KPZ scaling [2]. Also the string field theory formulation of the matrix model [3], which is supposed to be equivalent to 2d gravity coupled to $c = 1$ matter, describes a two dimensional field and thus presumably is the string field theory for a critical string in two target space dimensions. However many issues remain unresolved. The purpose of this letter is to highlight these questions and discuss some answers.

Let us consider the relation between critical and non-critical string theory. The latter, i.e. 2d gravity coupled to matter ($\underline{\theta}$), has the partition function

$$Z = \int \frac{[dg][d\theta]_g}{V_{diff}} e^{-I_M(\theta,g) - \lambda \chi - \int \mu_0 \sqrt{g}} \tag{1}$$

To be completely general at this stage we will not require I_M to be a conformal field theory. Following [4] we formulate the theory in the conformal gauge $g = e^\phi \hat{g}$. Then the partition function for the theory is given by

$$Z = \int [dX d(gh)]_{\hat{g}} e^{-I(X,\hat{g}) - I(gh)} \tag{2}$$

A priori in the above we must write the most general sigma model action

$$I(X,\hat{g}) = \frac{1}{8\pi} \int \sqrt{\hat{g}} \hat{g}^{\alpha\beta} G_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu + 2\sqrt{\hat{g}} \hat{R} \Phi(X) + \sqrt{\hat{g}} T(X) + ... \tag{3}$$

where $\mu, \nu, = 1, \ldots d$, $X^1, \ldots, X^{d-1} \equiv \underline{\theta}$, $\phi \equiv X^d$, and the ellipses in the action refer to possible non-renormalizable terms. $I(gh)$ is the ghost action.

The only obvious constraint on (3) follows from the fact (seen from (1)) that Z should be independent of the gauge fixing (invariant under $\hat{g} \rightarrow e^\rho \hat{g}$). This

means that the action I defines a conformal field theory. This statement translates into the requirement that the beta functions associated with the background fields $(G, \phi, T$ etc.) vanish. These beta functions are

$$\beta_{\mu\nu} = R_{\mu\nu} + 2\nabla_\mu\nabla_\nu\phi - \nabla_\mu T\nabla_\nu T + \ldots$$

$$\beta_\Phi = -R + 4(\nabla\Phi)^2 - 4\nabla^2\Phi + (\nabla T)^2 + V(T) + \frac{d-26}{3} + \ldots \qquad (4)$$

$$\beta_T = -2\nabla^2 T + 4\nabla\Phi\nabla T + V'(T) + \ldots$$

where the ellipses indicate higher derivative terms, higher powers of the tachyon, etc. The potential $V(T) = -2T^2 + \frac{T^3}{24} + \ldots^\star$ These beta function equations may be derived as equations of motion from the target space effective action

$$S = \int \sqrt{G}e^{-2\Phi}[-R - 4(\nabla\Phi)^2 + (\nabla T)^2 + V(T) + \frac{d-26}{3}] \qquad (5)$$

The graviton and dilaton terms in these equations were derived in [5]. The tachyonic contributions to the graviton and dilaton beta functions cannot be seen at any finite order in the loop expansion (the same is true of the tachyon potential $V(T)$) and were first computed in [6] (see also [7],[8]).

The assumption made in [4] is that the exponential form of the cosmological constant term in (1) is unchanged, and all that happens in passing from (1) to (2) is that the Liouville field gets rescaled and a dilaton linear in ϕ is generated. The two constants are determined by the requirement that the total central charge be zero and that the exponential tachyon be a (1,1) operator. But these are of course nothing but the conditions that the linearized beta functions vanish, i.e. the ansatz

$$\Phi = \frac{Q}{2}\phi; \quad G_{\mu\nu} = \delta_{\mu\nu}; \quad T = e^{\gamma\phi} \qquad (6)$$

satisfies (4) if we neglect $O(T^2)$ terms, provided that $Q^2 = \frac{25-(d-1)}{3}$ and that $\gamma^2 - Q\gamma + 2 = 0$. These are precisely the DDK [4] conditions.

$\star$ There are ambiguities in the higher order terms which will not concern us. See [9-11].

Now from the weak field expansion expressions (4) it seems as if the Liouville theory background cannot be an exact solution of the beta function conditions. However it can be shown [12] that, if one interprets a background independent version of the Das-Jevicki action as an effective action for the tachyon, then the Liouville action indeed defines a conformal field theory. In particular it also implies that there is no back reaction of the Liouville tachyon background on the metric. On the other hand, there is a second solution of the linearized equation, $\phi e^{\gamma \phi}$, which is not exact, though of course it could be the asymptotic form of an exact solution. This solution will have back reaction on the metric. Does such a solution have a non-critical string interpretation? More generally, one may take the black hole solution [13] perturbed by a tachyon background. The question of how the original solution gets modified by the tachyon is itself of interest and we will discuss that in this paper. But we may also ask whether there is a non-critical string interpretation; in particular what happens to KPZ scaling?

To examine these questions further we will first briefly discuss the derivation of the beta function equations. The most transparent and illuminating way to derive these is to use the local form of the Wilson-Polchinski [14] renormalization group equation.[†] We split the action (3) into a free and an interaction piece $I = I_0 + I_1$ (by writing $G_{\mu\nu} = \delta_{\mu\nu} + h_{\mu\nu}$) and then introduce an inverse Greens function G_ϵ into I_0 which is cutoff at short distance ϵ so as to regularize the theory. There are two sources of Weyl (or more generally world sheet two metric $\hat{g}$ dependence) in (2). One is the explicit dependence of I_1 and the other is through the dependence of the cut-off Greens function on $\hat{g}$. By a small generalization of the argument in [14] one then has the following equation:

$$\sqrt{\hat{g}} T^\alpha_\alpha(\sigma) = \frac{d - 26}{24\pi} \sqrt{\hat{g}} \hat{R}(\sigma) + \frac{\delta I_1}{\delta \rho(\sigma)}$$
$$+ \frac{1}{2} \frac{\delta G(\sigma_1, \sigma_2)}{\delta \rho(\sigma)} \left[\frac{\delta^2 I_1}{\delta X(\sigma_1) \delta X(\sigma_2)} - \frac{\delta I_1}{\delta X(\sigma_1)} \frac{\delta I_1}{\delta X(\sigma_2)} \right] \quad (7)$$

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† The fact that the beta function equations can be derived from this equation was first pointed out by Banks and Martinec [15].

where $\delta\rho$ is a conformal variation of the metric $\hat{g}$ and T_α^α is the trace of the 2d stress tensor. Note that the above is an operator equation in the theory defined by (2). Weyl invariance means the vanishing of the lhs and the various beta function equations are obtained by equating the coefficients of the independent operators $\sqrt{\hat{g}}, \sqrt{\hat{g}}\hat{R}, \sqrt{\hat{g}}\hat{g}^{\alpha\beta}\partial_\alpha X\partial_\beta X$, etc. to get the beta functions for respectively the tachyon, the dilaton, the graviton and higher mass states. In deriving these equations it is clear that one has to use the operator product expansion to write the second term inside the square brackets in terms of local operators. This as far as we know can only be done in the weak field expansion, so we can only evaluate these beta functions order by order in the coupling functions. However regardless of how the ope is evaluated it is clear from the structure of the equation that the quadratic term will correspond to the string interaction. So schematically, writing $I_1 = \sum g^i V_i$ (thus $g^i = T(X), G_{\mu\nu}(X) - \delta_{\mu\nu}, \ldots$) and $T_\alpha^\alpha = \sum \beta^i V_i$, we have the equations

$$\beta^i = \gamma_j^i g^j + c_{jk}^i(g)g^k g^l \tag{8}$$

In the above γ is the anomalous dimension matrix and $c_{jk}^i(g)$ are the ope coefficients. Note that since they are in general g dependent the string field theory interaction implied by (7) is not necessarily cubic. What the equation does seem to imply is that each beta function has all possible terms which are allowed by the fusion rules. In particular it would mean that in general there would be graviton-dilaton contributions to the beta function equations of higher spin states as well as that of the tachyon. This in turn could imply that if one turns on a non-trivial metric-dilaton background one would also be forced to turn on backgrounds of all other modes (which are allowed by the fusion rules) as well. An example of this is a four derivative contribution to the tachyon beta function [8,10] which is proportional to $R_{\lambda\mu\nu\sigma}^2$. This means that in a curved metric-dilaton background the tachyon must necessarily take a non-zero value.

Let us now consider the critical string point of view for two target space dimensions and see how the black hole solution [13, 16] is modified when other back-

grounds get turned on. In this case we may consider the beta function equations (4) which are indeed consistent with an effective action and incorporate the effects of the back reaction on the metric. The general solution of (4) (in the two derivative approximation) with zero tachyon background was obtained by Mandal et al [16]. This solution is in fact the low energy approximation to the gauged WZW theory considered by Witten and has been interpreted by him as a $1 + 1$ dimensional black hole. These observations have opened up the possibility of studying the string theoretic analogs of the black holes of classical general relativity, at least in two dimensional space-times. Indeed if the matrix model interpretation of these solutions could be elucidated then one might also be able to understand the full quantum theory of these stringy black holes.

As a first step toward this one needs an understanding of the non-critical string theory interpretation of these black hole solutions. This would require a study of what happens to these solutions when a tachyon background is turned on. It is of course a daunting task to solve the coupled beta function equations (even in the low energy approximation). However we believe that it is still possible to obtain some insight by studying the solutions in the asymptotic region, i.e. far away from the black hole horizon. Then we may ask whether the type of arguments made by Polchinski for the usual $1 + 1$ dimensional theory to get KPZ scaling (with log corrections as in the matrix model) remain valid. The affirmative answer to this question shows that there should be a non-critical string/matrix model interpretation of these general critical string backgrounds, even though it goes beyond the usual interpretation of the non-critical theory as the DDK one.

We will follow the method of Mandal et al [16] to obtain our modified black hole solution. In a local neighbourhood excluding critical points of Φ we may put $\Phi = \frac{Q}{2}\phi$ and choose the timelike coordinate θ orthogonal to ϕ so that the metric takes the form $G_{\mu\nu} = diag[G_{\theta\theta}, G_{\phi\phi}]$. We will work with a Euclidean metric. The Minkowski case can be obtained by putting $\theta \rightarrow it$.

In two dimensions, from the first equation in (4) we have the relation

$$\nabla_{\mu\nu}\Phi - \frac{1}{2}G_{\mu\nu}\nabla^2\Phi = \frac{1}{2}(\nabla_\mu T\nabla_\nu T - \frac{1}{2}G_{\mu\nu}\nabla T\nabla T) \tag{9}$$

Let us look for a solution with a static tachyon $T = T(\phi)$. With the above forms for the metric, dilaton and the tachyon, we have for the $\theta\theta, \theta\phi$, and $\phi\phi$ components of (9) the following equations:

$$\frac{Q}{8}\frac{G_{\theta\theta}}{G_{\phi\phi}}\partial_\phi\ln(G_{\theta\theta}G_{\phi\phi}) = -\frac{1}{4}\frac{G_{\theta\theta}}{G_{\phi\phi}}(T'(\phi))^2$$

$$-\frac{Q}{4}\partial_\theta\ln G_{\phi\phi} = 0$$

$$-\frac{Q}{8}\partial_\phi\ln(G_{\phi\phi}G_{\theta\theta}) = \frac{1}{4}(T'(\phi))^2$$

The general solution of these equations is

$$G_{\phi\phi} = \frac{1}{f(\phi)} \qquad G_{\theta\theta} = f(\phi)\exp[-\frac{2}{Q}\int^\phi (T'(\phi))^2]$$

Where $f(\phi)$ is an arbitrary function, and we have set an arbitrary function of θ in $G_{\theta\theta}$ to 1, by a coordinate transformation.

Calculating the curvature we find,

$$R^\theta_{\phi\theta\phi} = -\frac{1}{2}\left(\frac{h''}{h} - \frac{1}{2}\left(\frac{h'}{h}\right)^2\right) - \frac{1}{2}\left(\frac{f''}{f}\right) - \frac{3}{4}\frac{h'f'}{hf} \tag{10}$$

where we have defined

$$h \equiv \exp[-\frac{2}{Q}\int^\phi (T'(\phi))^2]$$

The graviton beta function gives one more equation which determines $f(\phi)$.

$$-\frac{1}{2}\left(\frac{f''}{f} - Q\frac{f'}{f}\right) = T'(T' - \frac{2}{Q}T'') + O(T^3) \tag{11}$$

The other two beta function equations give us the following additional relations.

From $\beta_T = 0$ we have,

$$T'' + \frac{f'}{f}T' + \frac{1}{2}\frac{h'}{h}T' - QT' + 2T - \frac{T^2}{8} + O(T^4) = 0 \tag{12}$$

and from $\beta_\Phi = 0$ we have,

$$Q^2 f - \frac{Q}{2}(2f' + f\frac{h'}{h}) - 2T^2 + \frac{T^3}{12} + O(T^4) - 8 = 0 \tag{13}$$

The three beta functions obey a Bianchi type identity[*]

$$\frac{1}{2}\nabla_\nu\beta_\Phi = \nabla^\mu\beta_{\mu\nu} - 2\nabla^\mu\Phi\beta_{\mu\nu} + \frac{1}{2}\nabla_\nu T\beta_T$$

This relation tells us that if the graviton and the tachyon equations are satisfied then the dilaton equation is automatically satisfied up to a constant. Thus (13) just determines the constant Q. We look for asymptotically flat solutions

$$f \rightarrow 1 \qquad T \rightarrow 0$$

as $\phi \rightarrow 0$ where we have rescaled ϕ to set the asymptotic value of f to one. Thus we have $Q^2 = 8$ exactly as in the flat space case.

We are able to solve the equation for the tachyon only in the asymptotic regime where we may ignore $O(T^2), O(\frac{f'}{f}T)$ terms compared to $O(T)$ terms. Then we get the linearized equation

$$T'' - QT' + 2T = 0$$

which has the familiar solutions (using the solution of the dilaton equation $Q = $

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[*] This identity is expected to hold for the exact beta functions not just for approximate ones given in (4). For the graviton dilaton system this was proved in perturbation theory by Curci and Pafutti [17]. A general proof independent of perturbation theory was given by Polchinski (quoted in [18]).

$-2\sqrt{2}$)

$$T = (\mu + \mu'\phi)e^{-\sqrt{2}\phi}$$

The first term is of course the Liouville solution. However unlike in Liouville theory where we have the exact tachyon field, from the *critical* string theory point of view the above is merely an asymptotic approximation to the exact tachyon-unless the interactions carefully arrange themselves as in the Das-Jevicki theory [12]. Furthermore, unlike in the standard interpretation of the non-critical string where there is no back reaction, in our case there is back reaction through equation (11). Let us compute this in the case where μ' is zero. Of course if the Das-Jevicki action is truely the action for the tachyon, then there is no back reaction in this case, and we should proceed immediately to the other case: $\mu' \neq 0$. However the results in either case are qualitatively the same, as will be seen below.

For $\mu' = 0$ the rhs of (11) is zero so that f satisfies

$$f'' - Qf' = 0$$

Thus with the above value for Q, and for f in the asymptotic region,

$$f = 1 - ae^{-2\sqrt{2}\phi} \tag{14}$$

With $h = 1$ this is Mandal et al's [16] form for the black hole solution and a is an arbitrary constant which is related to the mass of the black hole. For us this solution is modified since with the above asymptotic form for T

$$h = 1 - \frac{\mu^2}{2}e^{-2\sqrt{2}\phi}$$

to the same asymptotic order as f. Thus the black hole metric perturbed by the

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static tachyon is (with Minkowski signature)

$$ds^2 \simeq -(1 - \frac{\mu^2}{2}e^{-2\sqrt{2}\phi})(1 - ae^{-2\sqrt{2}\phi})dt^2 + (1 - ae^{-2\sqrt{2}\phi})^{-1}d\phi^2 \qquad (15)$$

When $\mu \to 0$ this solution tends to the form of the black hole solution given in [16].

What do we learn from this calculation? Our calculation is certainly not valid in the region of large curvature, but we might expect the general form fh for the time-time component and $1/f$ for the space-space component to remain valid, in which case what we have are approximate forms for f and h. Then what we've got (for $a > \mu^2/2$) is a solution with an outer horizon at $\phi = \frac{\ln a}{2\sqrt{2}}$ and an inner horizon at $\phi = \frac{\ln(\mu^2/2)}{2\sqrt{2}}$, reminiscent of the Reissner-Nordstrom black hole in four dimensions. However unlike in the case of the latter, where dt^2 and dx^2 flip sign together as either horizon is crossed, in our case when the the inner horizon is crossed only dt^2 flips sign. If $\mu^2 > a$ then the inner and outer horizons are interchanged. In either case the black hole character of the solution is preserved. Thus it would seem that turning on the tachyon does not affect the basic character of the solution. At this stage this is not a rigorous demonstration since we have only looked at the leading asymptotic behaviour, and it would be interesting to see whether this conclusion holds up under an analysis (perhaps numerical) of the coupled non-linear equations. Nevertheless we believe that it is a strong hint that the black hole solution is not destabilized by a tachyonic background. In addition, it should be remembered that what we are doing is perturbing in a marginal direction from Witten's black hole [13]. It is an important check on the physical significance of the solution that it is not of measure zero in the moduli space of conformal field theories.

The curvature for our modified solution can be calculated and we find

$$R = (\frac{\mu^2}{2} + a)8e^{-2\sqrt{2}\phi} \qquad (16)$$

This is singular at $\phi = -\infty$ as with the original black hole solution. Of course our approximations break down for $\phi \ll 0$ so we cannot really say with certainty that

the black hole singularity is undisturbed by the tachyon background. Note that this issue is not trivial. Indeed for the "negative mass" ($a<0$) case where we have a naked singularity* (16) indicates that the curvature vanishes and the black hole *disappears* for $\mu^2 = -a$. In our approximation of course this statement only makes sense in so far as the leading term in the asymptotic expansion vanishes. But this may be viewed as an indication that, in contrast to the black hole case, the naked singularity may be destabilized by turning on the tachyon background.

For completeness let us also write down the solution in the case where the tachyon has the more general form (14) with $\mu' \neq 0$:

$$
\begin{aligned}
f &= 1 - (a + a'\phi)e^{-2\sqrt{2}\phi} \\
g &= 1 - (b + b'\phi + b''\phi^2)e^{-2\sqrt{2}\phi}
\end{aligned}
\tag{17}
$$

b is a linear combination of $\mu^2, \mu'^2, \mu\mu'$. The only qualitative difference from the previous case is that the outer horizon (defined by the vanishing of f) gets shifted, unlike in the $\mu' = 0$ case.

What we have investigated in some detail here is the perturbation of the original black hole solution by just the tachyon background. Indeed as we discussed in the previous section a non-zero curvature solution necessitates a non-zero tachyon background. However (as asserted in [13] and [19]) this is but a special case of the more general (infinite parameter) background that may be generated by turning on the higher spin states of string theory (which in 1+1 dimensions exist only as discrete states [20]). Indeed the arguments we gave earlier (after equation (7)) may necessitate the turning on of these backgrounds. However provided these backgrounds vanish asymptotically, we expect, as in the purely tachyonic case, that the black hole character of the solution will not be affected. Thus what one has is a black hole with infinite hair. The "no-hair" theorem stating that a black hole is uniquely characterized by its mass, charge, and angular momentum is

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★ This kind of naked singularity is analogous to Witten's black hole as observed from regions V or VI of his Kruskal diagram[13].

related to the existence of Gauss law constraints coming from gauge invariance. In string theory one expects an infinite set of Gauss law constraints corresponding to the infinite number of gauge invariances. The propagating states do not of course cross the horizon from inside the hole, but sources of static gauge fields such as electric charge can be detected via the Gauss law. The discrete states in 2d string theory are presumably of this nature and so will correspond to different types of hair.

The Hawking temperature associated with the black hole solution is easily obtained from the metric (15) by comparing with the solution obtained directly from the gauged $SL(2,R)/U(1)$ WZW model [13, 21]. By comparing the two expressions for the dilaton we find the required coordinate transformation,

$$\frac{Q}{2}\phi = -\ln\cosh r + \frac{1}{2}\ln a$$

Then we have

$$ds^2 = \left(\frac{2}{Q}\right)^2 [(1 - \frac{\mu^2}{2}a^{-1}\cosh^{-2} r)\tanh^2 r d\tau^2 + dr^2]$$

In the above we have also defined the Euclidean time variable $\tau = \frac{2}{Q}\theta$.[*] This is the θ of [13] and from its origin as a parameter on the $SL(2,R)$ group manifold one knows that it ranges over $[0, 2\pi]$. Hence we can read off the Hawking temperature from the above and we find

$$T_H = \frac{|Q|}{4\pi} \tag{18}$$

Clearly the result is independent not only of the particular tachyonic background we have considered but also of any background which is asymptotically vanishing.[†] Furthermore as we argued in the discussion after (13) the value of Q is independent

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[*] Note that this comparison indicates that one must change $k \rightarrow k - 2$ in formula (9) of reference 13 in accordance with [21].

[†] A different conclusion was reached in [22], which however employed a different approximation scheme.

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of the background provided only that it vanishes asymptotically. Thus the full infinite parameter generalization of the black hole will have the same temperature (18) with $Q = -2\sqrt{2}$. This is quite unlike the case of 4d black holes in general relativity where the temperature is proportional to "surface gravity" which in the Schwartzschild case is just the inverse mass.

For the case of $c = 1$ matter coupled to 2d gravity the analyses of KPZ and DDK [23,4] failed to give the logarithmic corrections observed in the matrix model [24]. This issue was resolved by Polchinski [2] and by Das and Jevicki [3]. The latter argument was based on constructing the collective field theory for the corresponding matrix model and since its relation to the critical string theory is still unclear we shall say no more about it. The former on the other hand was directly based on interpreting the non-critical theory as a two dimensional critical string theory. We shall use this argument to show that KPZ scaling (with logarithmic corrections) applies even when the back reaction of the tachyon is taken into account as we have done in the previous section.

The translation invariance of the action (5) implies that if $G_{\mu\nu}, T(\phi), \Phi(\phi)$ are solutions of the field equations then so are

$$\overline{G}_{\mu\nu}(\phi) = G_{\mu\nu}(\phi - \overline{\phi}), \quad \overline{T}(\phi) = T(\phi - \overline{\phi}), \quad \overline{\Phi} = \Phi(\phi - \overline{\phi})$$

Note that in the above we have suppressed the dependence on θ. As before we may choose $\Phi = \frac{Q}{2}\phi$ and we get the asymptotic solution for the β_T equation (4): $T(\phi) \rightarrow b(\phi)e^{-\sqrt{2}(\phi)}$ for $\phi \rightarrow \infty$. The latter limit from the non-critical string theory standpoint corresponds to taking the short distance limit on the world sheet $(g = e^{-\sqrt{2}\phi}\hat{g})$ and following Polchinski we introduce a cutoff at $\phi = 0$ and define the bare world sheet cosmological constant as

$$\overline{T}(0, \theta) = \Delta$$

This gives (for $\overline{\phi}$ large and negative and Δ small)

$$-b\overline{\phi}e^{\sqrt{2}\,\overline{\phi}} = \Delta$$

Now Polchinski also needed (in order to insure finiteness of the integral for the tachyon free energy for large negative ϕ) the behaviour $T(\phi) \to 1 - b'e^{(2-\sqrt{2})\phi}$ as $\phi \to -\infty$ which would follow from the β_T equation if the metric were flat and the constant part of T were unambiguously determined by (4). But we know that because of the back reaction the metric cannot remain flat. However in the case of the Euclidean black hole, even with the tachyon background (and possibly other backgrounds) the lower limit of the integral is automatically cut off (as observed by Witten in a different context) at what would turn out to be the horizon if we continued the solution to Minkowski space. The rest of the argument for KPZ scaling with log corrections follow as in [2]. Similarly one can derive the scaling relations for the two point function [25].

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